International Journal of Engineering & Scientific Research
Vol.5 Issue 12, December 2017,
ISSN: 2347-6532 Impact Factor: 6.660
Journal Home page: http://www.ijmra.us, Email:editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

DOUBLE DIFFUSIVE CONVECTION OF POROUS MATRIX SATURATED WITH NANOFLUID

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Abstract

The onset of double diffusive convection in a horizontal porous medium saturated by a nanofluid with Soret and Dufour effects has been studied using both linear and nonlinear stability analysis. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis, while the modified Darcy model has been used for the porous medium that includes the time derivative term to model the momentum equation. The thermal energy equations include the diffusion and cross diffusion terms. The linear theory depends on normal mode technique and nonlinear analysis depends on a minimal representation of double Fourier series. The effects of Soret and Dufour parameters, Solutal Rayleigh number on the stationary and oscillatory convections are presented graphically. It is found that for stationary mode Soret parameter and Dufour parameter have a stabilizing effect while Solutal Rayleigh number destabilize the system and for oscillatory mode Soret parameter, Dufour parameter have a stabilizing effect while Solutal Rayleigh number destabilize the system. We also study the effect of time on transient Nusselt numbers which is found to be oscillatory when time is small. However, when time becomes very large all the three transient Nusselt values approaches to the steady value.

Keywords:
Nanofluid, porous medium, natural convection, horizontal layer, conductivity and viscosity variation, Brownian motion and thermophoresis.

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1. Introduction

The study of nanofluid has become increasingly important in the last few decades. This is mainly due to their applications in electronics, automotive, high flux solar collectors, and nuclear applications where improved heat transfer or efficient heat dissipation is required Wong and Leon [1]. Nanofluid is a dilute suspension of solid nanoparticles (1–100 nm diameters) in conventional liquids like water, ethylene or tri-ethylene-glycols, and oil. Depending on size, shape, and thermal properties of the nanoparticles,the thermal conductivity can be increased by about 40% with low concentration (1%–5% by volume) of solid nanoparticles in the mixture Eastman et al [2]. Buongiorno [3] deduced the governing equations for nanofluid incorporating the effects of Brownian diffusion and thermophoresis. The problem of double diffusive convection in porous media has attracted considerable interest during the last few decades because of its wide range of applications, from the solidification of binary mixtures to the migration of solutes in water-saturated soils, geophysical systems, electrochemistry and the migration of moisture through air contained in fibrous insulation.

A comprehensive review of the literature concerning double diffusive convection in a fluid-saturated porous medium may be found in the book by Nield and Bejan[4]. The study of double diffusive convection in porous medium is first under taken by Nield [5] on the basis of linear stability theory for various thermal and solutal boundary conditions.

Excellent reviews of these studies have been reported by Turner [6-8], Huppert and Turner [9] and Platten and Legros [10]. The interest in the study of two or multi component convection has developed as a result of the marked difference between single component and multi component systems. In contrast to single component system, convection sets is even when density decreases with height, that is, when the basic state is hydrostatically stable. The double diffusive convection is of importance in various fields such as high quality crystal production, liquid gas storage, oceanography, production of pure medication, solidification of molten alloys, and geothermally heated lakes and magmas.

In a system where two diffusing properties are present, instabilities can occur only if one of the components are destabilizing. When heat and mass transfer occur simultaneously in a moving

fluid, the relation between the fluxes and the driving potentials are of more intricate in nature. It has been found that an energy flux can be generated not only by temperature gradient but also by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this is the Soret or thermal-diffusion effect. If the cross-diffusion terms are included in the species transport equations, then the situation will be quite different. Due to the cross-diffusion effects, each property gradient has a significant influence on the flux of the other property. The double diffusive convection in a porous medium in the presence of Soret and Dufour coefficients has been analyzed by Rudraiah and Malashetty [11] and extended to weak non-linear analysis by Rudraiah and Siddheshwar [12].

Many studies on the thermal instability of nanofluids have been conducted by Kuznetsov and Nield [13] and Nield and Kuznetsov [14]. Kuznetsov and Nield [15] studied the onset of thermal instability in a porous medium saturated by a nanofluid using Brinkman model, and incorporating the effects of Brownian motion and thermophoresis of nanoparticles. They concluded that the critical thermal Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy, by the presence of the nanoparticles. The corresponding Horton–Rogers–Lapwood Problem was investigated by Nield and Kuznetsov [14] for the Darcy Model. Nield and Kuznetsov Agarwal et al. [16] studied thermal instability in an anisotropic rotating porous layer saturated by a nanofluid for top heavy and bottom heavy suspension considering Darcy Model. Bhadauria and Agarwal [17] studied natural convection in a rotating porous layer saturated by a nanofluid using the Brinkman's Model.

For the preparation of nanofluids, instead of using a pure liquid as basefluid, when a binary liquid is used, it is termed as a binary nanofluid. These binary liquids can be salty water, Ferro fluid etc. These binary nanofluids find their utility as a working fluid in absorption refrigeration, as a solution in electro or electro less plating and as a transfer medium in medical treatment. Onset of convection in binary fluids has been studied by Kim et al. [18], Kuznetsov and Nield [19] and Kuznetsov and Nield [13]. In case of nanofluids, we come across two

different Soret effects: one induced by the solute, while the other being contributed by the nanoparticles.

Using the approach employed by Tiwari and Das [20], we tend to investigate the contribution of both Soret and Dufour effects in convection in binary nanofluids. The important objective of this study is to perform nonlinear stability analysis of the problem using the minimal representation of Fourier series to compute heat and mass transports.

2. Analysis

2.1 Conservation Equation for a Nanofluid

We select a coordinate frame in which the *z*-axis is aligned vertically upwards. We consider a horizontal layer of fluid confined between the planes z* = 0 and z* = *H*. Asterisks are used to denote dimensional variables. Each boundary wall is assumed to be perfectly thermally conducting. The temperatures at the lower and upper boundary are taken to be $T_0^* + \Delta T^*$ and T^*. The Oberbeck Boussinesq approximation is employed. In the linear stability theory being applied here, the temperature change in the fluid is assumed to be small in comparison with T_0^*. The conservation equation takes the form

$$\nabla^* . \mathbf{v}_D^* = 0 \tag{1}$$

Here, $\mathbf{v}_D^*$ is the nanofluid Darcy velocity. We write $\mathbf{v}_D^* = (u^*, v^*, w^*)$.

In the presence of thermophoresis, the conservation equation for the nanoparticles, in the absence of chemical reactions, takes the form

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon}\mathbf{v}_D^* . \nabla \phi^* = \nabla^* . \left[D_B \nabla^* \phi^* + D_T \frac{\nabla^* T^*}{T^*} \right] \tag{2}$$

where ϕ^* is the nanoparticle volume fraction, ε is the porosity, T^* is the temperature, D_B is the Brownian diffusion coefficient, and D_T is the thermophoretic diffusion coefficient.

If one introduces a buoyancy force and adopts the Boussinesq approximation, and uses the Darcy model for a porous medium, then the momentum equation can be written as

$$\frac{\rho}{\varepsilon}\frac{\partial \mathbf{v}_D^*}{\partial t^*} = -\nabla^* p^* - \frac{\mu_{eff}}{K}\mathbf{v}_D^* + \rho \mathbf{g} \tag{3}$$

Here ρ is the overall density of the nanofluid, which we now assume to be given by

$$\rho = \phi^* \rho_{\mathrm{p}} + \left(1-\phi^*\right)\rho_0\left[1-\beta_{\mathrm{T}}\left(T^*-T_0^*\right)-\beta_C\left(C^*-C_0^*\right)\right]$$

(4)

where ρ_{p} is the particle density, ρ_0 is a reference density for the fluid, and β_{T} is the thermal volumetric expansion coefficient and β_C is the analogous solutal coefficient. The thermal energy equation for a nanofluid can be written as

$$\left(\rho c\right)_{\mathrm{m}} \frac{\partial T^*}{\partial t^*} + \left(\rho c\right)_{\mathrm{f}} \mathbf{v}_{\mathrm{D}}^*.\nabla^* T^* = k_{\mathrm{m}} \nabla^{*2} T^* + \varepsilon\left(\rho c\right)_{\mathrm{p}}\left[D_{\mathrm{B}} \nabla^* \phi^*.\nabla T^* + D_{\mathrm{T}} \frac{\nabla T^*.\nabla T^*}{T_0^*}\right] + \rho c\, D_{TC}\, \nabla^{*2} C^* \qquad (5)$$

Here c is the fluid specific heat (at constant pressure), k_m is the overall thermal conductivity of the porous medium saturated by the nanofluid, c_p is the nanoparticle specific heat of the material constituting the nanoparticles, D_{TC} is a diffusivity of Dufour type.

To this we add a conservation equation for the solute of the form

$$\frac{\partial C^*}{\partial t^*} + \frac{1}{\varepsilon} V^*{}_D.\nabla^* C^* = D_{Sm} \nabla^{*2} C^* + D_{CT} \nabla^{*2} T^*$$

(6)

Where D_{Sm} is the solutal diffusivity for the porous medium and D_{CT} is a diffusivity of Soret type. It has been assumed that the nanoparticles do not affect the transport of the solute.

Thus,

$$k_m = \varepsilon k_{eff} + (1-\varepsilon) k_s$$

(7)

where ε is the porosity, k_{eff} is the effective conductivity of the nanofluid (fluid plus nanoparticles), and k_s is the conductivity of the solid material forming the matrix of the porous medium.

We now introduce the viscosity and the conductivity dependence on nanoparticle fraction. Following Tiwari and Das [20], we adopt the formulas, based on a theory of mixtures,

$$\frac{\mu_{eff}}{\mu_f} = \frac{1}{(1-\phi^*)^{2.5}}$$

(8)

$$\frac{k_{eff}}{k_f} = \frac{(k_p + 2k_f) - 2\phi^*(k_f - k_p)}{(k_p + 2k_f) + \phi^*(k_f - k_p)}$$

(9)

Here k_f and k_p are the thermal conductivities of the fluid and the nanoparticles, respectively.

Equation (8) was obtained by Brinkman [21], and (9) is the Maxwell-Garnett formula for a suspension of spherical particles that dates back to Maxwell [22].

In the case where ϕ^* is small compared with unity, we can approximate these formulas by

$$\frac{\mu_{eff}}{\mu_f} = 1 + 2.5\phi^*, \qquad \frac{k_{eff}}{k_f} = \frac{(k_p + 2k_f) - 2\phi^*(k_f - k_p)}{(k_p + 2k_f) + \phi^*(k_f - k_p)} = 1 + 3\phi^* \frac{(k_p - k_f)}{(k_p + 2k_f)}$$

(10)

We assume that the temperature and the volumetric fraction of the nanoparticles are constant on the boundaries. Thus the boundaries conditions are

$$w^* = 0,\ T^* = T_0^* + \Delta T^*, \phi^* = \phi_0^* \quad \text{at} \quad z^* = 0, \qquad w^* = 0,\ T^* = T_0^*, \phi^* = \phi_1^* \quad \text{at} \quad z^* = H$$

(11)

We introduce dimensionless variables as follows. We define

$$(x, y, z) = (x^*, y^*, z^*)/H,\ t = t^*\alpha_m/\sigma H^2,\ (u, v, w) = (u^*, v^*, w^*)H/\alpha_m,\ p = p^*K/\mu_f\alpha_m,$$

$$\phi = \frac{\phi^* - \phi_0^*}{\phi_1^* - \phi_0^*}, \qquad T = \frac{T^* - T_0^*}{\Delta T^*}$$

(12)

where

$$\alpha_m = \frac{k_m}{(\rho c_p)_f}, \quad \sigma = \frac{(\rho c_p)_m}{(\rho c_p)_f}$$

We also define

$$\tilde{\mu} = \frac{\mu_{eff}}{\mu_f}, \qquad k = \frac{k_p}{k_f}, \qquad \tilde{k}_s = \frac{k_s}{k_f}, \qquad \tilde{k} = \frac{k_m}{k_f}$$

(13)

From (7), (10) and (13) we have

$$\tilde{\mu} = 1 + 2.5[\phi_0^* + \phi(\phi_1^* - \phi_0^*)], \qquad \tilde{k} = \varepsilon\left\{1 + 3[\phi_0^* + \phi(\phi_1^* - \phi_0^*)]\frac{\tilde{k}_p - 1}{\tilde{k}_p + 2}\right\} + (1 - \varepsilon)\tilde{k}_s$$

(14)

Then (1) and (3) with (4), (5), (2), (11) take the form:

$$\nabla.\mathbf{v} = 0$$

(15)

$$\gamma_a \frac{\partial \mathbf{v}}{\partial t} = -\nabla p - \tilde{\mu}\,\mathbf{v} - Rm\hat{e}_z + Ra_T T\hat{e}_z + (Rs/Le)C\hat{e}_z - Rn\phi\hat{e}_z$$

(16)

$$\frac{\partial T}{\partial t} + \mathbf{v}.\nabla T = \tilde{k}\nabla^2 T + \frac{N_B}{Ln}\nabla\phi.\nabla T + \frac{N_A N_B}{Ln}\nabla T.\nabla T + N_{TC}\nabla^2 C$$

(17)

$$\frac{1}{\sigma}\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}\mathbf{v}.\nabla C = \frac{1}{Le}\nabla^2 C + N_{CT}\nabla^2 T$$

(18)

$$\frac{1}{\sigma}\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{v}.\nabla\phi = \frac{1}{Ln}\nabla^2\phi + \frac{N_A}{Ln}\nabla^2 T$$

(19)

$$w = 0,\ T = 1,\ C = 1,\ \phi = 0 \ \ at\ z = 0, \qquad w = 0,\ \ T = 0,\ C = 0,\ \phi = 0 \ at\ z = 1$$

(20)

Here

$$\gamma_a = \frac{\varepsilon}{\sigma Va},\ Ln = \frac{\alpha_m}{D_B},\ Va = \frac{\varepsilon^2\,\mathrm{Pr}}{Da},\ Ra_T = \frac{\rho g \beta_T K H \Delta T^*}{\mu_f \alpha_m},\ \mathrm{Pr} = \frac{\mu_f}{\rho\,\alpha_m},\ Da = \frac{K}{H^2},\ Rs = \frac{\rho g \beta_C K H \Delta C^*}{\mu_f\, D_{Sm}},$$

$$Rm = \frac{\left[\rho_p \phi_0^* + \rho(1-\phi_0^*)\right] gKH}{\mu_f \alpha_m},\ Rn = \frac{(\rho_p - \rho)(\phi_1^* - \phi_0^*) gKH}{\mu_f \alpha_m},\ N_A = \frac{D_T \Delta T^*}{D_B T_c^* (\phi_1^* - \phi_0^*)},\qquad N_B = \frac{\varepsilon(\rho c)_p (\phi_1^* - \phi_0^*)}{(\rho c)_f},$$

$$Le = \frac{\alpha_f}{D_S},\ N_{TC} = \frac{D_{TC}\Delta C^*}{\alpha_m \Delta T^*},\ N_{CT} = \frac{D_{CT}\,\Delta T^*}{\alpha_m \Delta C^*}$$

The parameter γ_a is the non dimensional acceleration coefficient, Ln is a thermo-nanofluid Lewis number, Va is a Vadász number, Ra_T is the familiar thermal Rayleigh–Darcy number, Pr is the Prandtl number, *Da* is the Darcy number, *Rs* is the familiar solutal Rayleigh number, The new parameters *Rm* and *Rn* may be regarded as a basic-density Rayleigh number and a concentration Rayleigh number, respectively. The parameter N_A is a modified diffusivity ratio and is some what similar to the Soret parameter that arises in cross-diffusion phenomena in solutions, while N_B is a modified particle-density increment, *Le* is the familiar thermo-solutal Lewis number,

In the spirit of the Oberbeck–Boussinesq approximation, (16) has been linearized by the neglect of a term proportional to the product of ϕ and T. This assumption is likely to be valid in the case of small temperature gradients in a dilute suspension of nanoparticles.

2.2. Basic solution

We seek a time-independent quiescent solution of (15)–(20) with temperature and nanoparticle volume fraction varying in the z-direction only that is a solution of the form

$$\mathbf{v}=0,\ p=p_b(z),\ T=T_b(z),\ C=C_b(z),\ \phi=\phi_b(z)$$

(21)

Equations (16)-(19) reduces to

$$0=-\frac{dp_b}{dz}-Rm+Ra_T\,T_b+\left(Rs/Le\right)C_b-Rn\,\phi_b$$

(22)

$$\tilde{k}\frac{d^2T_b}{dz^2}+\frac{N_B}{Ln}\frac{d\phi_b}{dz}+\frac{N_AN_B}{Ln}\left(\frac{dT_b}{dz}\right)^2+N_{TC}\frac{d^2C_b}{dz^2}=0$$

(23)

$$\frac{1}{Le}\frac{d^2C_b}{dz^2}+N_{CT}\frac{d^2T_b}{dz^2}=0$$

(24)

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0$$

(25)

According to Buongiorno [3], for most nanofluids investigated so far $Ln/\left(\phi_1^*-\phi_0^*\right)$ is large, of order 10^5–10^6, and since the nanoparticle fraction decrement is typically no smaller than 10^3 this means so that Ln is large, of order 10^2–10^3, while N_A is no greater than about 10. Using this approximation, the basic solution is found to be

$$T_b=1-z,\qquad C_b=1-z\qquad \text{and}\qquad \text{so}\qquad \phi_b=z$$

(26)

2.3. Perturbation solution

We now superimpose perturbations on the basic solution. We Write

$$\mathbf{v} = \mathbf{v}' \qquad , p = p_b + p', \qquad T = T_b + T', C = C_b + C', \qquad \phi = \phi_b + \phi'$$

(27)

substitute in (13)–(19), and linearize by neglecting products of primed quantities. The following equations are obtained when (26) is used.

$$\nabla.\mathbf{v}' = 0,$$

(28)

$$\gamma_a \frac{\partial \mathbf{v}'}{\partial t} = -\nabla p' - \tilde{\mu}\,\mathbf{v}' + Ra_T T'\hat{e}_z + (Rs/Le)C'\hat{e}_z - Rn\phi'\hat{e}_z,$$

(29)

$$\frac{\partial T'}{\partial t} - w' = \tilde{k}\nabla^2 T' + \frac{N_B}{Ln}\left(\frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z}\right) - \frac{2N_A N_B}{Ln}\frac{\partial T'}{\partial z} + N_{TC}\nabla^2 C',$$

(30)

$$\frac{1}{\sigma}\frac{\partial C'}{\partial t} + \frac{1}{\varepsilon}w' = \frac{1}{Le}\nabla^2 C + N_{CT}\nabla^2 T,$$

(31)

$$\frac{1}{\sigma}\frac{\partial \phi'}{\partial t} + \frac{1}{\varepsilon}w' = \frac{1}{Ln}\nabla^2\phi' + \frac{N_A}{Ln}\nabla^2 T'$$

(32)

$$w' = 0, T' = 0, C' = 0, \phi' = 0 \text{ at } z = 0 \text{ and at } z = 1,$$

(33)

where now we can approximate the viscosity and conductivity distributions by substituting the basic solution expression for ϕ, namely that given by (26), into (14), we obtain

$$\tilde{\mu}(z) = 1 + 2.5\left[\phi_0^* + \phi(\phi_1^* - \phi_0^*)z\right],\ \tilde{k}(z) = \varepsilon\left\{1 + 3\left[\phi_0^* + \phi\left(\phi_1^* - \phi_0^*\right)z\right]\frac{\tilde{k}_p - 1}{\tilde{k}_p + 2}\right\} + (1-\varepsilon)\tilde{k}_s$$

(34)

It will be noted that the parameter Rm is just a measure of the basic static pressure gradient and is not involved in these and subsequent equations.

We now recognize that we have a situation where properties are heterogeneous. These are now the viscosity and conductivity (rather that the more usual ones, namely permeability and conductivity) and we can now proceed as in a number of papers by the authors that are surveyed by Nield [23]. We assume that the heterogeneity is weak in the sense that the maximum variation

of a property over the domain considered is small compared with the mean value of that property.

The seven unknowns $u', v', w', p', T', C', \phi'$ can be reduced to four by operating on (29) with $\hat{e}_z$ curl curl and using the identity curl curl $\equiv$ grad div $-\nabla^2$ together with (28) and the weak heterogeneity approximation. The result is

$$\left(\tilde{\mu}(z)+s\gamma_a\right)\nabla^2 w' = Ra_T\nabla_H^2 T' + \left(Rs/Le\right)\nabla_H^2 C' - Rn\nabla_H^2\phi'$$

(35)

Here ∇_H^2 is the two-dimensional Laplacian operator on the horizontal plane.

The differential (35), (29), (30), (31), (32) and the boundary conditions (33) constitute a linear boundary-value problem that can be solved using the method of normal modes.

We write

$$\left(w', T', C', \phi'\right) = \left[W(z), \Theta(z), \Sigma(z), \Phi(z)\right]\exp\left(st + ilx + imy\right)$$

(36)

and substitute into the differential equations to obtain

$$\left(\tilde{\mu}(z)+s\gamma_a\right)\left(D^2-\alpha^2\right)W + Ra_T\,\alpha^2\,\Theta + \left(Rs/Le\right)\alpha^2\,\Sigma - Rn\,\alpha^2\,\Phi = 0$$

(37)

$$W + \left(\left(D^2-\alpha^2\right)\tilde{k}(z) + \frac{N_B}{Ln}D - \frac{2N_AN_B}{Ln}D - s\right)\Theta - \frac{N_B}{Ln}D\Phi + N_{TC}\left(D^2-\alpha^2\right)\Sigma = 0$$

(38)

$$\frac{1}{\varepsilon}W + \frac{1}{Le}\left(D^2-\alpha^2-\frac{1}{\sigma}s\right)\Sigma + N_{CT}\left(D^2-\alpha^2\right)\Theta = 0$$

(39)

$$\frac{1}{\varepsilon}W - \frac{N_A}{Ln}\left(D^2-\alpha^2\right)\Theta - \left(\frac{1}{Ln}\left(D^2-\alpha^2\right) - \frac{1}{\sigma}s\right)\Phi = 0$$

(40)

$W = 0,\ \Theta = 0, \Sigma = 0, \Phi = 0$ at $z = 0$ and $z = 1$

(41)

where

$D \equiv \frac{d}{dz}$ and $\alpha = (l^2 + m^2)^{1/2}$.

(42)

Thus α is a dimensionless horizontal wave number.

For neutral stability the real part of s is zero. Hence we now write $s = i\omega$, where ω is real and is a dimensionless frequency.

We now employ a Galerkin-type weighted residuals method to obtain an approximate solution to the system of (37)–(41). We choose as trial functions (satisfying the boundary conditions) $W_p, \Theta_p, \Sigma_p, \Phi_p ; p = 1, 2, 3.....$ and write

$$W = \sum_{p=1}^{N} A_p W_p \, , \Theta = \sum_{p=1}^{N} B_p \Theta_P \, , \Sigma = \sum_{p=1}^{N} C_p \Sigma_p \, , \Phi = \sum_{p=1}^{N} D_p \Phi_P$$

(43)

substitute into (37)–(41), and make the expressions on the left-hand sides of those equations (the residuals) orthogonal to the trial functions, thereby obtaining a system of 4N linear algebraic equations in the 4N unknowns A_p, B_p, C_p, D_p, p =1, 2, . . . N. The vanishing of the determinant of coefficients produces the eigenvalue equation for the system. One can regard Ra_T as the eigenvalue. Thus Ra_T is found in terms of the other parameters.

Trial functions satisfying the boundary condition (41) can be chosen as

$$W_p = \Theta_p = \Phi_p = \Sigma_p = \sin p\pi z \, ; p = 1, 2, 3, \ldots$$

(44)

The eigenvalue equation is

$$\det M = 0$$

(45)

where,

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}$$

(46)

and, for i, j = 1, 2, …, N.

$$(M_{11})_{ij} = -\left\langle \left(\tilde{\mu}(z) + \gamma_a s\right) W_j D^2 W_i \right\rangle + \alpha^2 \left\langle \left(\tilde{\mu}(z) + \gamma_a s\right) W_j W_i \right\rangle$$

$$(M_{12})_{ij} = -Ra_T \alpha^2 \langle W_j \Theta_i \rangle$$
$$(M_{13})_{ij} = -(Rs/Le)\alpha^2 \langle W_j \Sigma_i \rangle$$
$$(M_{14})_{ij} = Rn\alpha^2 \langle W_j \Phi_i \rangle$$

$$(M_{21})_{ij} = -\langle \Theta_j W_i \rangle$$

$$(M_{22})_{ij} = -\langle \tilde{k}(z)\Theta_j D^2 \Theta_i \rangle + \alpha^2 \langle \tilde{k}(z)\Theta_j \Theta_i \rangle + s\langle \Theta_j \Theta_i \rangle + \left(\frac{2N_A N_B}{Ln} - \frac{N_B}{Ln} \right) \langle \Theta_j D\Theta_i \rangle$$

$$(M_{23})_{ij} = -\langle N_{TC}\Theta_j D^2 \Sigma_i \rangle + \alpha^2 \langle N_{TC}\Theta_j \Sigma_i \rangle$$
$$(M_{24})_{ij} = \frac{N_B}{Ln} \langle \Theta_j D\Phi_i \rangle$$

$$(M_{31})_{ij} = -\frac{1}{\varepsilon} \langle \Sigma_j W_i \rangle$$
$$(M_{32})_{ij} = -\langle N_{CT}\Theta_i D^2 \Sigma_j \rangle + \alpha^2 \langle N_{CT}\Theta_i \Sigma_j \rangle$$
$$(M_{33})_{ij} = \frac{1}{Le}\left(-\langle \Sigma_j D^2 \Sigma_i \rangle + \alpha^2 \langle \Sigma_j \Sigma_i \rangle + \frac{s}{\sigma} \langle \Sigma_j \Sigma_i \rangle \right)$$
$$(M_{34})_{ij} = 0$$
$$(M_{41})_{ij} = -\frac{1}{\varepsilon} \langle \Phi_j W_i \rangle$$
$$(M_{42})_{ij} = \frac{N_A}{Ln}\left(\langle \Phi_j D^2 \Theta_i \rangle - \alpha^2 \langle \Phi_j \Theta_i \rangle \right)$$

$$(M_{43})_{ij} = 0$$

$$(M_{44})_{ij} = \frac{1}{Ln}\left(\langle \Phi_j D^2 \Phi_i \rangle - \alpha^2 \langle \Phi_j \Phi_i \rangle \right) - \frac{s}{\sigma} \langle \Phi_j \Phi_i \rangle$$

Here

$$\langle f(z) \rangle \equiv \int_0^1 f(z)\, dz. \qquad (47)$$

In the present case, where viscosity and conductivity variations are incorporated, the critical wavenumber is unchanged and the stability boundary becomes

$$Ra_T = \frac{1}{\alpha^2}\left(\frac{JN_{TC}}{\varepsilon}\left(\frac{J}{Ln}+\frac{s}{\sigma}\right)-\frac{1}{Le}\left(J+\frac{s}{\sigma}\right)\left(\frac{J}{Ln}+\frac{s}{\sigma}\right)\right)^{-1}$$
$$\left[\begin{array}{l}\left((-J)\left(\frac{J+s}{Le}\right)\left(\frac{J}{Ln}+\frac{s}{\sigma}\right)\left(J+\frac{s}{\sigma}\right)+J^3N_{TC}N_{CT}\left(\frac{J}{Ln}+\frac{s}{\sigma}\right)\right)(1+s\gamma_a)+\\ \frac{Rs\alpha^2}{Le}\left(-JN_{CT}\left(\frac{J}{Ln}+\frac{s}{\sigma}\right)+\left(\frac{J+s}{\varepsilon}\right)\left(\frac{J}{Ln}+\frac{s}{\sigma}\right)\right)+\\ Rn\alpha^2\left(\frac{N_A J}{Le\,Ln}\left(J+\frac{s}{\sigma}\right)+\left(\frac{J+s}{Le\,\varepsilon}\right)\left(J+\frac{s}{\sigma}\right)-JN_{TC}\left(\frac{N_A J}{\varepsilon\,Ln}+\frac{JN_{CT}}{\varepsilon}\right)\right)\end{array}\right]$$

(48)

where

$J = (\pi^2+\alpha^2)$, $\nu = 1+1.25\left(\phi_1^*+\phi_0^*\right)$

$$\eta = \varepsilon+(1-\varepsilon)\tilde{k}_s+\frac{3\left(\phi_1^*+\phi_0^*\right)\varepsilon}{2}\left(\frac{\tilde{k}_p-1}{\tilde{k}_p+2}\right)$$

(49)

We observe that when there is no conductivity variation (that is $\eta = 1$, as when $\tilde{k}_s = 1$ and $\tilde{k}_p = 1$) the effect of viscosity variation is to increase the critical Rayleigh number by a factor ν. The additional effect of conductivity variation η is expressed by (49). When $\tilde{k}_s = 1$, the maximum value of η is $2.5\left(\phi_1^*+\phi_0^*\right)$ attained when $\varepsilon = 1$ and $\tilde{k}_p \to \infty$.

It is worth noting that the factor ν comes from the mean value of $\tilde{\mu}(z)$ over the range [0,1] and the factor η is the mean value of $\tilde{k}(z)$ over the same range. That means that when evaluating the critical Rayleigh number it is a good approximation to base that number on the mean values of the viscosity and conductivity based in turn on the basic solution for the nanofluid fraction.

3. Linear Stability Analysis

3(a). Stationary Mode

For the validity of principle of exchange of stabilities (i.e., steady case), we have $s = 0$ $(i.e., s = s_r + is_i = s_r = s_i = 0)$ at the margin of stability. For a first approximation we take $N = 1$. Then the Rayleigh number at which marginally stable steady mode exists becomes,

$$Ra_T^{St} = \frac{1}{\alpha^2}\left(\frac{JN_{TC}}{\varepsilon}\left(\frac{J}{Ln}\right) - \frac{1}{Le}(J)\left(\frac{J}{Ln}\right)\right)^{-1}$$
$$\left[\begin{array}{l}\left((-J)\left(\frac{J}{Le}\right)\left(\frac{J}{Ln}\right)(J) + J^3 N_{TC} N_{CT}\left(\frac{J}{Ln}\right)\right) + \\ \frac{Rs\,\alpha^2}{Le}\left(-JN_{CT}\left(\frac{J}{Ln}\right) + \left(\frac{J}{\varepsilon}\right)\left(\frac{J}{Ln}\right)\right) + \\ Rn\,\alpha^2\left(\frac{N_A J}{Le\,Ln}(J) + \left(\frac{J}{Le\,\varepsilon}\right)(J) - JN_{TC}\left(\frac{N_A J}{\varepsilon\,Ln} + \frac{JN_{CT}}{\varepsilon}\right)\right)\end{array}\right]$$

(50)

In the case of double diffusion in a regular fluid, when N_{TC}, N_{CT} and N_A are all zero, (50) reduces to $Ra_T^{St} + Rs = R_0$, where ($R_0 = 4\pi^2 = 39.48$ with $\alpha_c = \pi = 3.14$)as expected. (See, for example, Nield [5].) The stationary boundary does not depend on the value of Pr.

3(b). Oscillatory Mode

We now set $s = i\omega$, where $\omega = \mathrm{Im}(\omega)(\omega_r = 0)$ in (48) and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\,\omega\,\Delta_2$$

(51)

For oscillatory onset $\Delta_2 = 0 \quad (\omega_i \neq 0)$ and this gives a dispersion relation of the form (on dropping the subscript *i*)

$$b_1\left(\omega^2\right)^2 + b_2\left(\omega^2\right) + b_3 = 0$$

(52)

Now Eq. (48) with $\Delta_2 = 0$ gives

$$Ra_T^{Osc} = a_0\left(a_1 + \omega^2 a_2\right)$$

(53)

where $b_1, b_2,$ and b_3 and $a_0, a_1,$ and a_2 and Δ_1 and Δ_2 are not presented here for brevity.

We find the oscillatory neutral solutions from (53). It proceeds as follows: First determine the number of positive solutions of (42). If there are none, then no oscillatory instability is possible. If there are two, then the minimum (over a^2) of (53) with ω^2 given by (52) gives the oscillatory neutral Rayleigh number. Since (52) is quadratic in ω^2, it can give rise to more than one positive

value of ω^2 for fixed values of the parameters *Rn, Ln,* N_A, σ, γ_a, and λ. However, our numerical solution of (52) for the range of parameters considered here gives only one positive value of ω^2 indicating that there exists only one oscillatory neutral solution. The analytical expression for oscillatory Rayleigh number given by (53) is minimized with respect to the wavenumber numerically, after substituting for ω^2 (> 0) from (52), for various values of physical parameters in order to know their effects on the onset of oscillatory convection.

4. Non – Linear stability analysis

For simplicity, we consider the case of two dimensional rolls, assuming all physical quantities to be independent of y. Eliminating the pressure and introducing the stream function we obtain:

$$(1+s\gamma_a)\nabla^2\Psi + Ra_T\frac{\partial T}{\partial x} + \left(\frac{Rs}{Le}\right)\frac{\partial C}{\partial x} - Rn\frac{\partial S}{\partial x} = 0$$

(54)

$$\frac{\partial T}{\partial t} + \frac{\partial \Psi}{\partial x} = \nabla^2 T + \frac{\partial(\Psi, T)}{\partial(x,z)} + N_{TC}\nabla^2 C$$

(55)

$$\frac{1}{\sigma}\frac{\partial S}{\partial T} + \frac{1}{\varepsilon}\frac{\partial \Psi}{\partial x} = \frac{1}{Ln}\nabla^2 S + \frac{N_A}{Ln}\nabla^2 T + \frac{1}{\varepsilon}\frac{\partial(\Psi, S)}{\partial(x,z)}$$

(56)

$$\frac{1}{\sigma}\frac{\partial C}{\partial T} + \frac{1}{\varepsilon}\frac{\partial \Psi}{\partial x} = \frac{1}{Le}\nabla^2 C + N_{CT}\nabla^2 T + \frac{1}{\varepsilon}\frac{\partial(\Psi, C)}{\partial(x,z)}$$

(57)

We solve (54)–(57) subjecting them to stress-free, isothermal, iso-nanoconcentration boundary conditions:

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = T = S = C = 0 \text{ at } z = 0, 1$$

To perform a local non-linear stability analysis, we take the following Fourier expressions:

$$\psi = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}(t)\sin(m\alpha x)\sin(n\pi z)$$

$$T = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{mn}(t)\cos(m\alpha x)\sin(n\pi z)$$

$$S = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{mn}(t)\cos(m\alpha x)\sin(n\pi z)$$

$$C = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{mn}(t)\cos(m\alpha x)\sin(n\pi z)$$

(58)

Further, we take the modes (1, 1) for stream function, and (0, 2) and (1, 1) for temperature, and nanoparticle concentration, to get

$$\psi = A_{11}(t)\sin(\alpha x)\sin(\pi z)$$

$$T = B_{11}(t)\cos(\alpha x)\sin(\pi z) + B_{02}(t)\sin(2\pi z)$$

$$S = C_{11}(t)\cos(\alpha x)\sin(\pi z) + C_{02}(t)\sin(2\pi z)$$

$$C = D_{11}(t)\cos(\alpha x)\sin(\pi z) + D_{02}(t)\sin(2\pi z)$$

(59)

where the amplitudes $A_{11}(t)$, $B_{11}(t)$, $B_{02}(t)$, $C_{11}(t)$, $C_{02}(t)$, $D_{11}(t)$ and $D_{02}(t)$ are functions of time and are to be determined.

Taking the orthogonality condition with the eigenfunctions associated with the considered minimal model, we get

$$\frac{dA_{11}(t)}{dt} = \frac{1}{\gamma_a \delta^2}\left[\alpha Rn\, C_{11}(t) - \alpha Ra_T\, B_{11}(t) - \frac{\alpha Rs\, D_{11}(t)}{Le} - \delta^2 A_{11}(t)\right]$$

$$\frac{dB_{11}}{dt} = -\left[\alpha A_{11}(t) + \delta^2 B_{11}(t) + \alpha\pi A_{11}(t) B_{02}(t) + N_{TC}\,\delta^2 D_{11}(t)\right]$$

$$\frac{dB_{02}}{dt} = -4\pi^2 B_{02}(t) + \frac{\alpha\pi}{2} A_{11}(t) B_{11}(t) - N_{TC}\, 4\pi^2 D_{02}(t)$$

$$\frac{dC_{11}}{dt} = -\sigma\left[\frac{1}{\varepsilon}\alpha A_{11}(t) + \delta^2\left(\frac{C_{11}(t)}{Ln} + \frac{N_A}{Ln} B_{11}(t)\right) + \frac{1}{\varepsilon}\alpha\pi A_{11}(t) C_{02}(t)\right]$$

$$\frac{dC_{02}}{dt} = -\sigma\left[\frac{1}{Ln} 4\pi^2 C_{02}(t) + 4\pi^2 B_{02}(t)\frac{N_A}{Ln} - \frac{a\pi}{2\varepsilon} A_{11}(t) C_{11}(t)\right]$$

$$\frac{dD_{11}}{dt} = -\sigma\left[\frac{1}{\varepsilon}\alpha A_{11}(t) + \delta^2\left(\frac{D_{11}(t)}{Le} + N_{CT} B_{11}(t)\right) + \frac{1}{\varepsilon}\alpha\pi A_{11}(t) D_{02}(t)\right]$$

$$\frac{dD_{02}}{dt} = -\sigma\left[\frac{1}{Le} 4\pi^2 D_{02}(t) + 4\pi^2 B_{02}(t) N_{CT} - \frac{a\pi}{2\varepsilon} A_{11}(t) D_{11}(t)\right]$$

(60)

In case of steady motion $\frac{d(\,)}{dt} = D_i = 0$, ($i$ = 1, 2, .., 7) and write all D_i's in terms of A_{11}.

Thus we get

$$D_1 = \frac{1}{\gamma_a \delta^2}\left[\alpha Rn\, C_{11}(t) - \alpha Ra_T\, B_{11}(t) - \frac{\alpha Rs\, D_{11}(t)}{Le} - \delta^2 A_{11}(t)\right]$$

$$D_2 = -\left[\alpha A_{11}(t) + \delta^2 B_{11}(t) + \alpha\pi A_{11}(t)\, B_{02}(t) + N_{TC}\,\delta^2 D_{11}(t)\right]$$

$$D_3 = -4\pi^2 B_{02}(t) + \frac{\alpha\pi}{2} A_{11}(t) B_{11}(t) - N_{TC}\, 4\pi^2 D_{02}(t)$$

$$D_4 = -\sigma\left[\frac{1}{\varepsilon}\alpha A_{11}(t) + \delta^2\left(\frac{C_{11}(t)}{Ln} + \frac{N_A}{Ln} B_{11}(t)\right) + \frac{1}{\varepsilon}\alpha\pi A_{11}(t) C_{02}(t)\right]$$

$$D_5 = -\sigma\left[\frac{1}{Ln} 4\pi^2 C_{02}(t) + 4\pi^2 B_{02}(t)\frac{N_A}{Ln} - \frac{a\pi}{2\varepsilon} A_{11}(t) C_{11}(t)\right]$$

$$D_6 = -\sigma\left[\frac{1}{\varepsilon}\alpha A_{11}(t) + \delta^2\left(\frac{D_{11}(t)}{Le} + N_{CT} B_{11}(t)\right) + \frac{1}{\varepsilon}\alpha\pi A_{11}(t) D_{02}(t)\right]$$

$$D_7 = --\sigma\left[\frac{1}{Le} 4\pi^2 D_{02}(t) + 4\pi^2 B_{02}(t) N_{CT} - \frac{a\pi}{2\varepsilon} A_{11}(t) D_{11}(t)\right]$$

And $D_1 = D_2 = D_3 = D_4 = D_5 = D_6 = D_7 = 0$ (61)

The above system of simultaneous autonomous ordinary differential equations is solved numerically using Runge–Kutta–Gill method. One may also conclude that the trajectories of the above equations will be confined to the finiteness of the ellipsoid. Thus, the effect of the parameters *Rn*, *Ln*, N_A on the trajectories is to attract them to a set of measure zero, or to a fixed point to say.

5. Heat and Nanoparticle Concentration Transport

The Thermal Nusselt number NuT is defined as

$$NuT = \frac{\text{Heat transport by (conduction + convection)}}{\text{Heat transport by conduction}}$$

$$= 1 + \left[\frac{\int_0^{2\pi} \frac{\partial T}{\partial z}\, dx}{\int_0^{2\pi/a} \frac{\partial T_B}{\partial z}\, dx}\right]_{z=0}$$

Substituting expressions (26) and (59) in above equation we get

$$NuT = 1 - 2\pi B_{02}(t)$$

The nanoparticle concentration Nusselt number NuF is defined similar to the thermal Nusselt number. Following the procedure adopted for arriving at NuT, one can obtain the expression for NuF in the form:

$$NuF = \left(1-2\pi C_{02}(t)\right) + N_A\left(1-2\pi B_{02}(t)\right)$$

The solute concentration Nusselt number NuC is defined similar to the thermal Nusselt number. Following the procedure adopted for arriving at NuT, one can obtain the expression for NuC in the form:

$$NuC = \left(1-2\pi D_{02}(t)\right) + N_{CT}\left(1-2\pi B_{02}(t)\right)$$

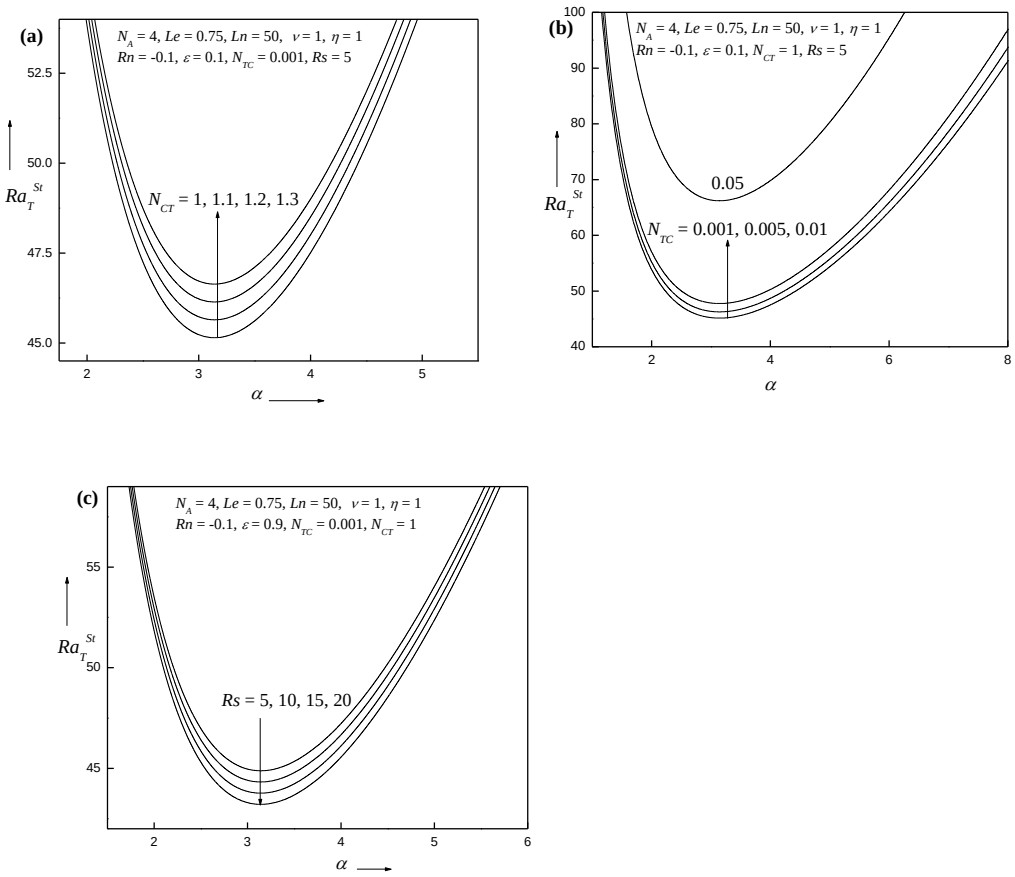


Figure 1. Neutral curves on stationary convection for different values of **(a)** Soret parameter N_{CT}**, (b)** Dufour parameter N_{TC}**, (c)** Solutal Rayleigh number *Rs***.**

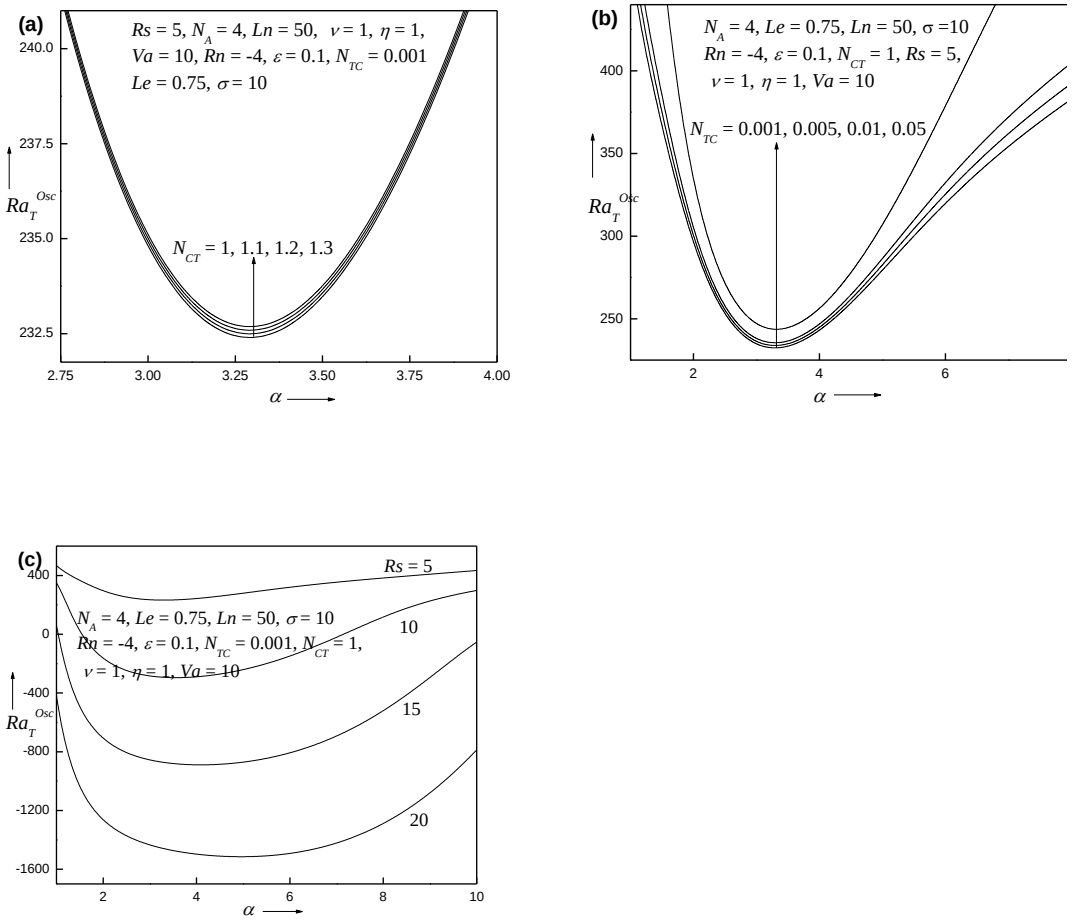
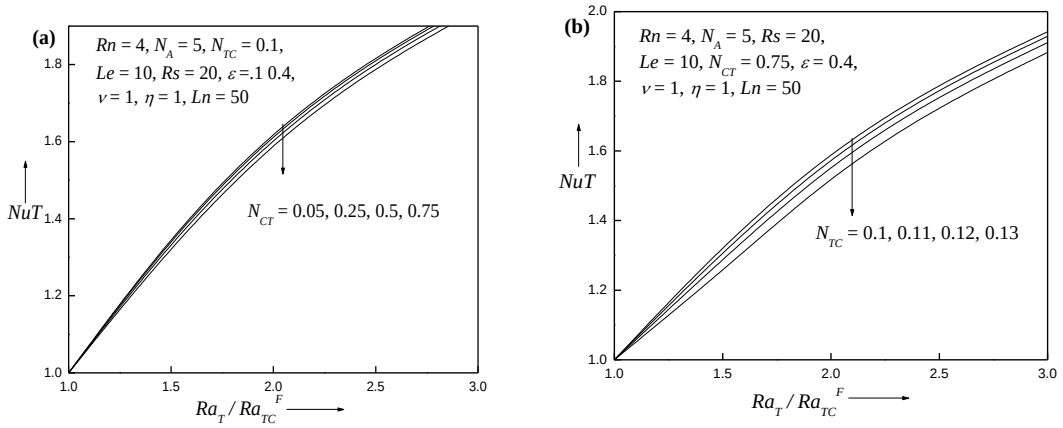


Figure 2. Neutral curves on oscillatory convection for different values of **(a)** Soret parameter N_{CT} **, (b)** Dufour parameter N_{TC} **, (c)** Solutal Rayleigh number *Rs***.**



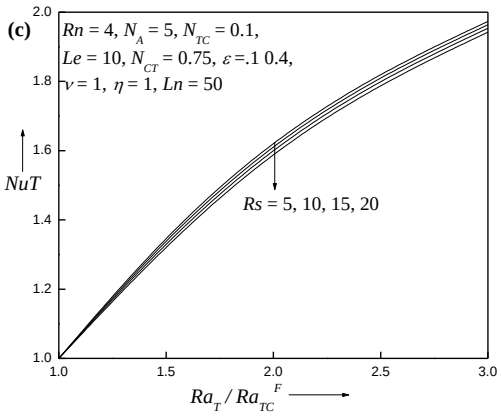


Figure 3. Variation of Thermal Nusselt number *NuT* with critical Rayleigh Number for different values of **(a)** Soret parameter N_{CT} **, (b)** Dufour parameter N_{TC} **, (c)** Solutal Rayleigh number *Rs*.

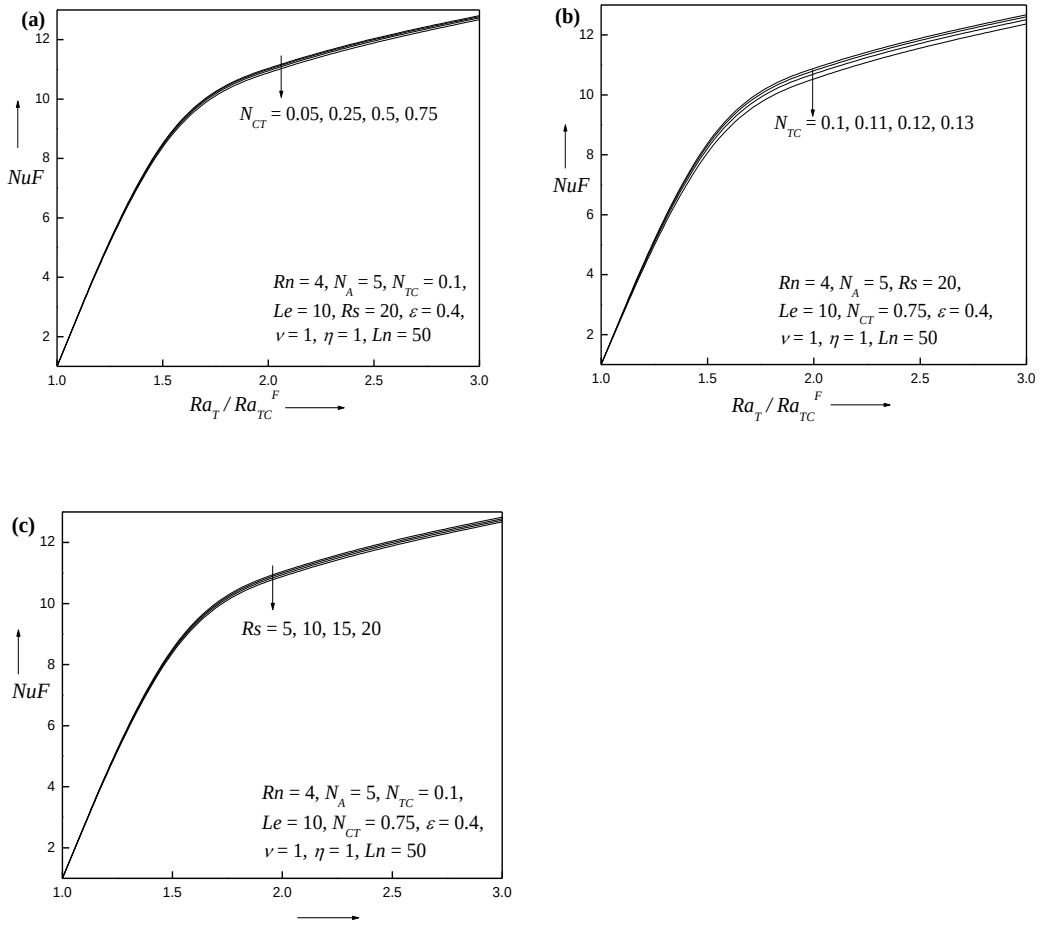


Figure 4. Variation of nanoparticle concentration Nusselt number *NuF* with critical Rayleigh Number for different values of **(a)** Soret parameter N_{CT} **, (b)** Dufour parameter N_{TC} **, (c)** Solutal Rayleigh number *Rs*.

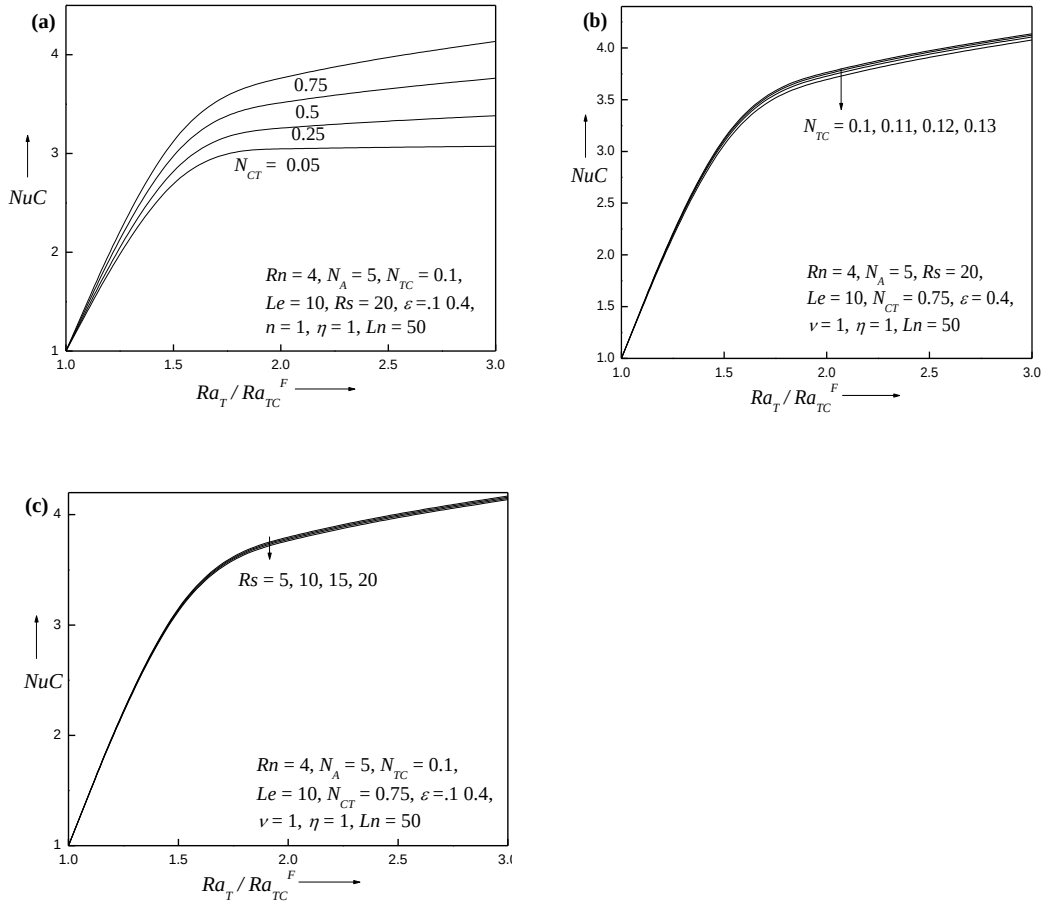
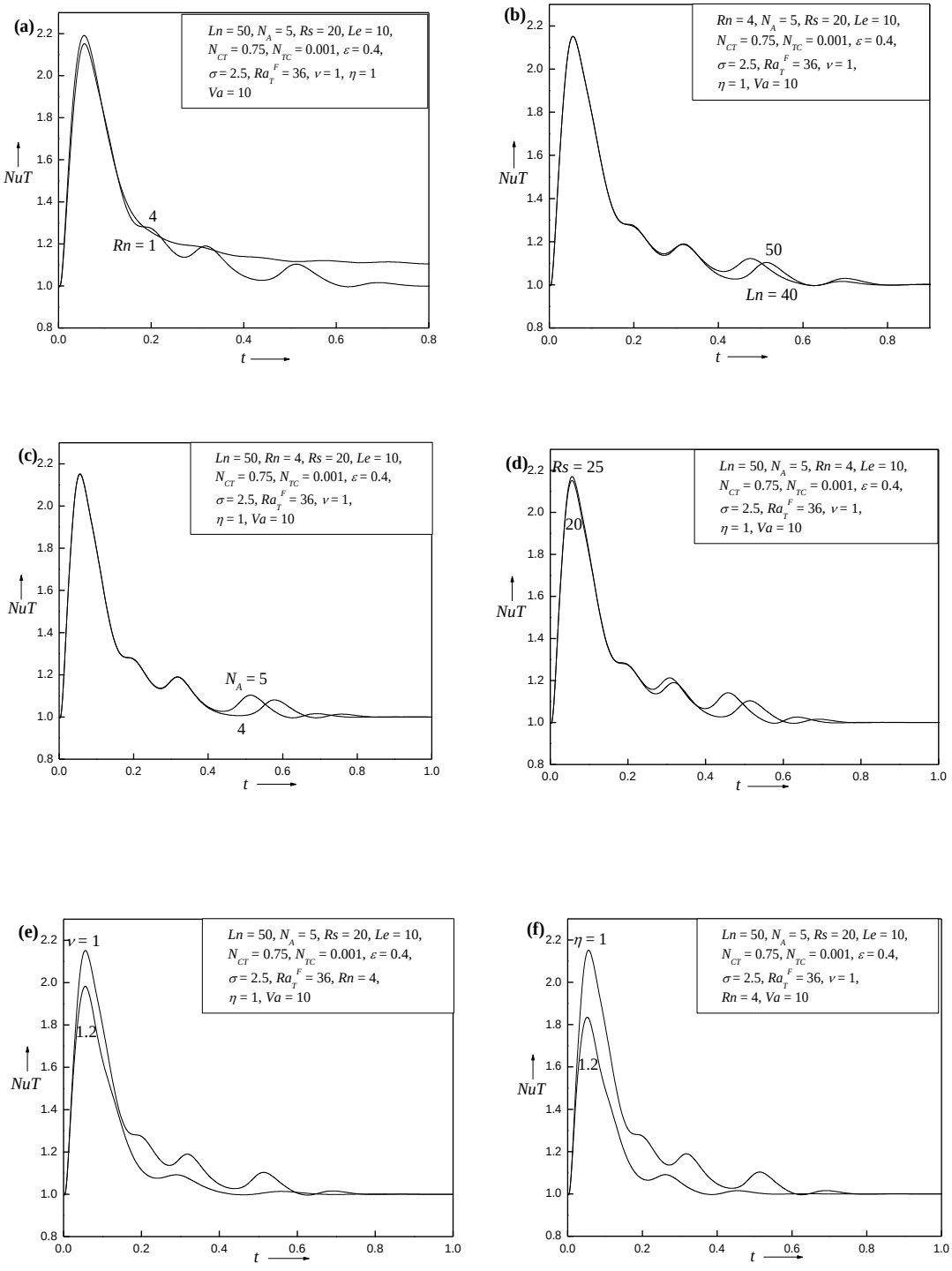


Figure 5. Variation of solute concentration Nusselt number *NuC* with critical Rayleigh Number for different values of **(a)** Soret parameter N_{CT} **, (b)** Dufour parameter N_{TC} **, (c)** Solutal Rayleigh number *Rs*.


(a)
Ln = 50, N_A = 5, Rs = 20, Le = 10,
N_CT = 0.75, N_TC = 0.001, ε = 0.4,
σ = 2.5, Ra_T^F = 36, ν = 1, η = 1
Va = 10
4
Rn = 1
NuT
t
(b)
Rn = 4, N_A = 5, Rs = 20, Le = 10,
N_CT = 0.75, N_TC = 0.001, ε = 0.4,
σ = 2.5, Ra_T^F = 36, ν = 1,
η = 1, Va = 10
50
Ln = 40
NuT
t
(c)
Ln = 50, Rn = 4, Rs = 20, Le = 10,
N_CT = 0.75, N_TC = 0.001, ε = 0.4,
σ = 2.5, Ra_T^F = 36, ν = 1,
η = 1, Va = 10
N_A = 5
4
NuT
t
(d)
Rs = 25
Ln = 50, N_A = 5, Rn = 4, Le = 10,
N_CT = 0.75, N_TC = 0.001, ε = 0.4,
σ = 2.5, Ra_T^F = 36, ν = 1,
η = 1, Va = 10
20
NuT
t
(e)
ν = 1
Ln = 50, N_A = 5, Rs = 20, Le = 10,
N_CT = 0.75, N_TC = 0.001, ε = 0.4,
σ = 2.5, Ra_T^F = 36, Rn = 4,
η = 1, Va = 10
1.2
NuT
t
(f)
η = 1
Ln = 50, N_A = 5, Rs = 20, Le = 10,
N_CT = 0.75, N_TC = 0.001, ε = 0.4,
σ = 2.5, Ra_T^F = 36, ν = 1,
Rn = 4, Va = 10
1.2
NuT
t

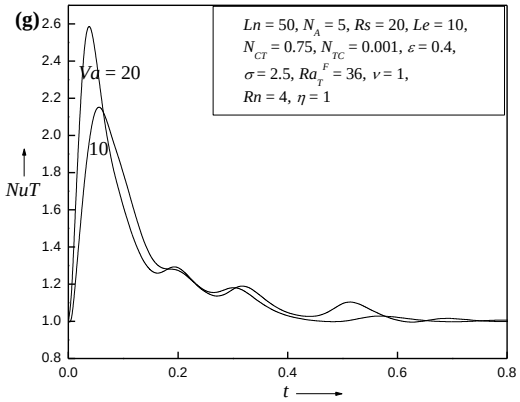
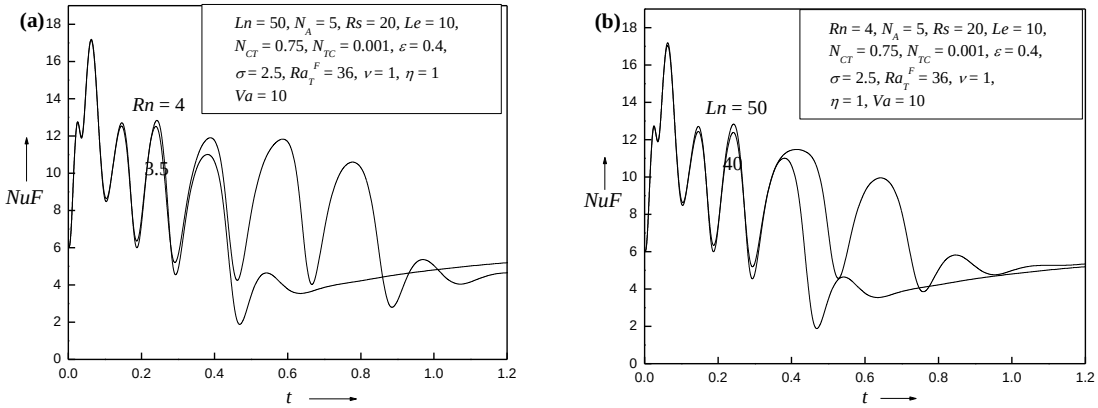


Figure 6. Transient Thermal Nusselt number *NuT* with time for different values of **(a)** Nanoparticle concentration Rayleigh number *Rn*, **(b)** Thermo-nanofluid Lewis number *Ln*, **(c)** Modified diffusivity ratio N_A, **(d)** Solutal Rayleigh number *Rs*, **(e)** Viscosity ratio ν, **(f)** Conductivity ratio η, **(g)** Vadász number *Va*.



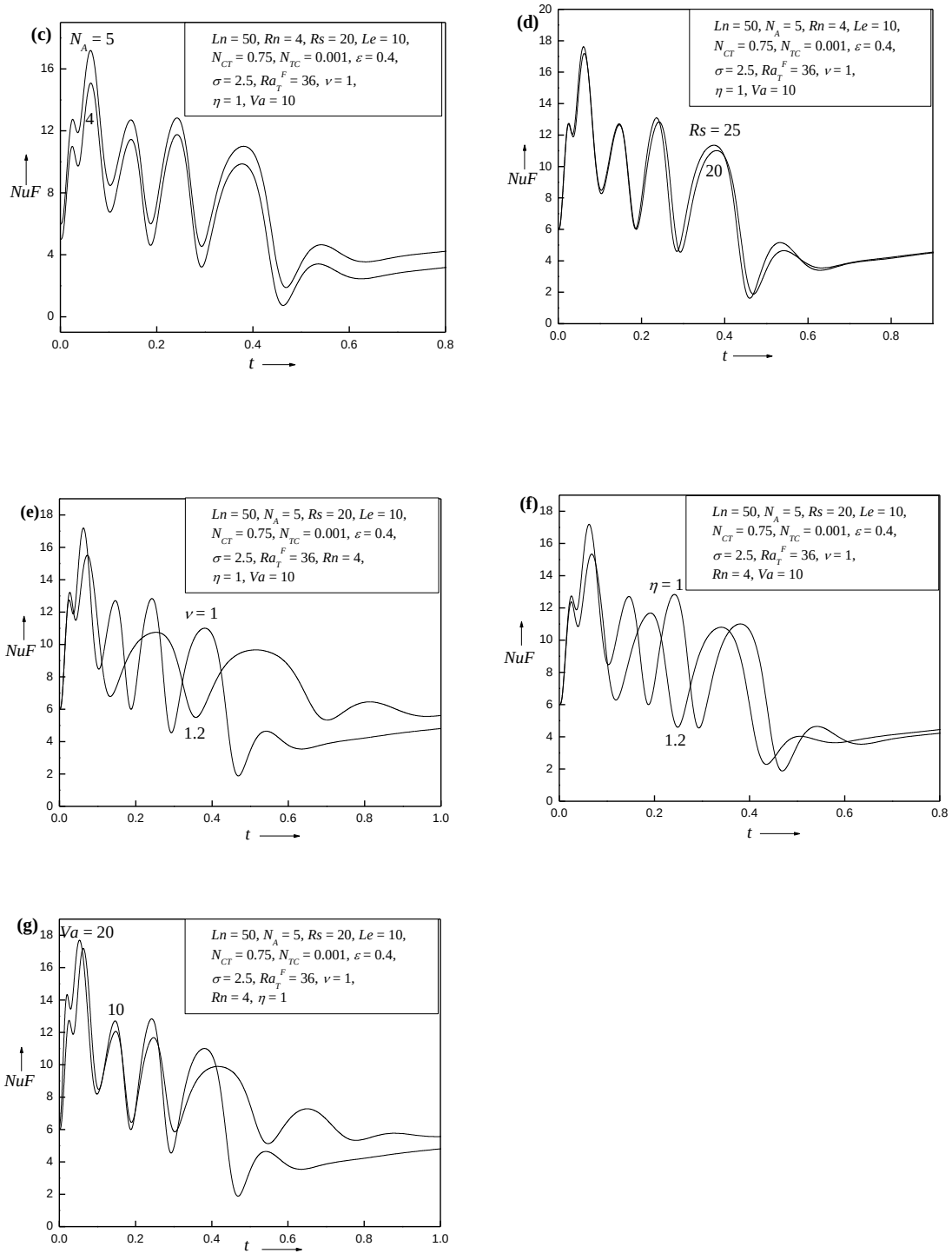
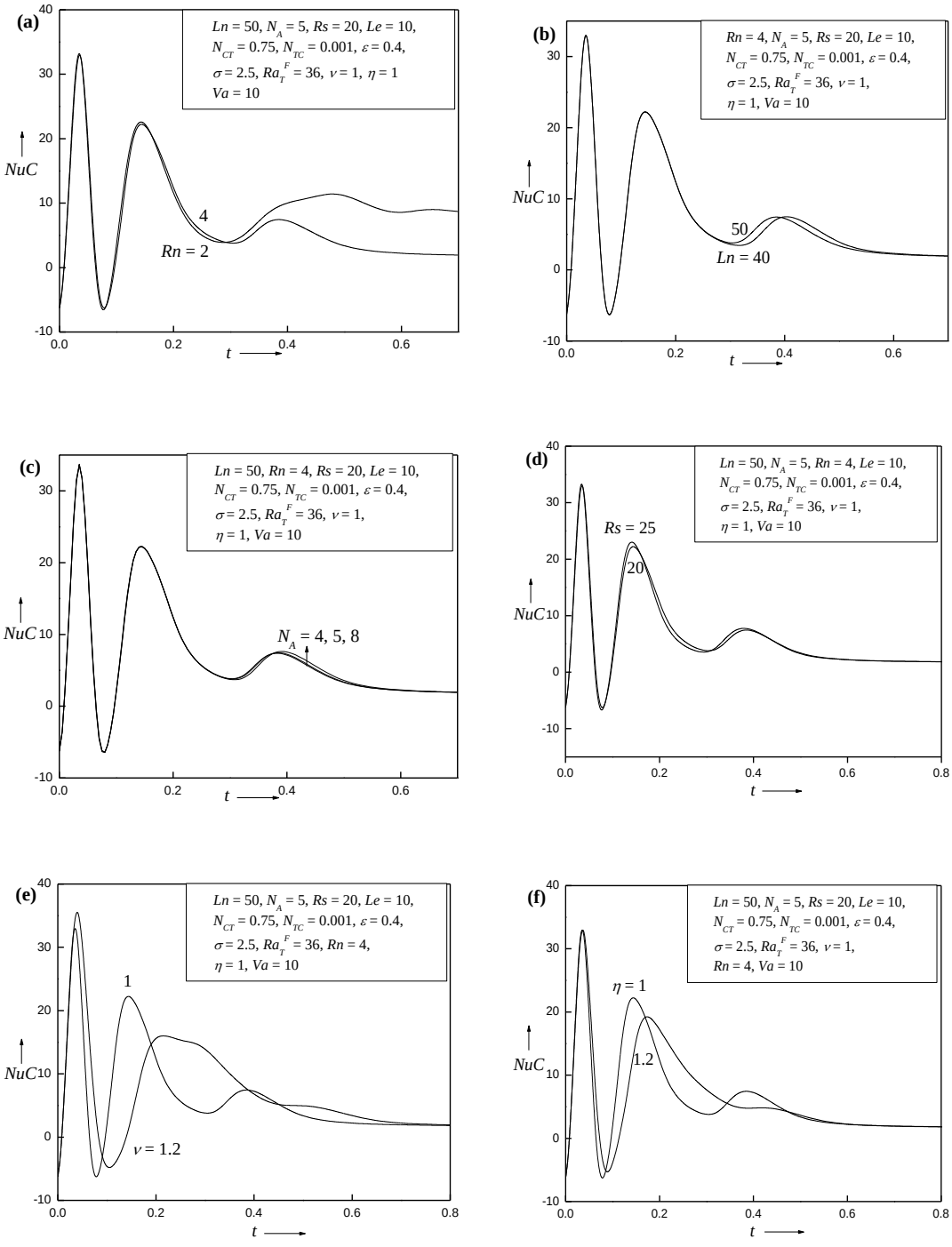


Figure 7. Transient nanoparticle concentration Nusselt number *NuF* with time for different values of **(a)** Nanoparticle concentration Rayleigh number *Rn*, **(b)** Thermo-nanofluid Lewis

number *Ln*, **(c)** Modified diffusivity ratio N_A, **(d)** Solutal Rayleigh number *Rs*, **(e)** Viscosity ratio ν, **(f)** Conductivity ratio η, **(g)** Vadász number *Va*.



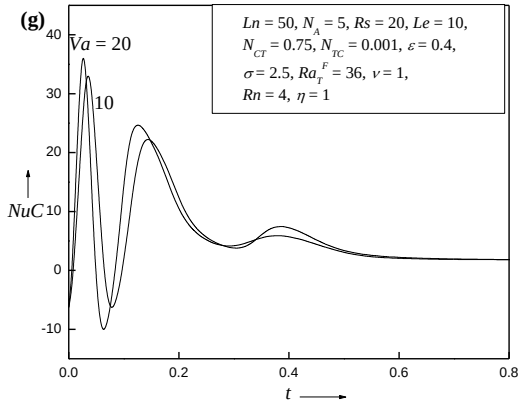


Figure 8. Transient solute concentration Nusselt number *NuC* with time for different values of **(a)** Nanoparticle concentration Rayleigh number *Rn*, **(b)** Thermo-nanofluid Lewis number *Ln*, **(c)** Modified diffusivity ratio N_A, **(d)** Solutal Rayleigh number *Rs*, **(e)** Viscosity ratio ν, **(f)** Conductivity ratio η, **(g)** Vadász number *Va*.

6. Results and discussions

The expressions of thermal Rayleigh number for stationary and oscillatory convections are given by (50) and (53) respectively.Figure 1a-c shows the effect of various parameters on the neutral stability curves for stationary convection for *Rn* = -0.1, *Ln* = 50, $N_A = 4$, $\varepsilon = 0.9$, $\nu = 1$, $\eta = 1$, $N_{TC} = 0.001$, *Rs* = 5, *Le* = 0.75, $N_{CT} = 1$ with variation in one of these parameters. The effect of Soret parameter N_{CT} and Dufour parameter N_{TC} on the thermal Rayleigh number is shown in Figs . 1a and 1b respectively, it can be seen that as N_{CT} and N_{TC} increases Ra_T increases and hence N_{CT} and N_{TC} have a stabilizing effect on the system. From Fig. 1c, one can observe that as Solutal Rayleigh number *Rs* increases, thermal Rayleigh number decreases which means that the Solutal Rayleigh number *Rs* advances the onset of convection. The effect of Soret parameter N_{CT}, Dufour parameter N_{TC} and Solutal Rayleigh number *Rs* on thermal Rayleigh number Ra_T for stationary convection show the similar results obtained by Agarwal et al. [24].

Figure 2a-c displays the variation of thermal Rayleigh number for oscillatory convection with respect to various parameters. The effect of Soret parameter N_{CT} and Dufour parameter N_{TC} on the thermal Rayleigh number is shown in Figs. 2a and 2b respectively, it is seen that as N_{CT} and

N_{TC} increases Ra_T increases and hence N_{CT} and N_{TC} have a stabilizing effect on the system. From Fig. 2c, one can observe that as Solutal Rayleigh number *Rs* increases, thermal Rayleigh number decreases which means that the Solutal Rayleigh number *Rs* advances the onset of convection.

The nonlinear analysis provides not only the onset threshold of finite amplitude motion but also the information of heat and mass transports in terms of thermal Nusselt number *NuT*, nanoparticle concentration Nusselt number *NuF* and solute concentration Nusselt number *NuC* Nusselt numbers. The Nusselt numbers are computed as the functions of Ra_T, and the variations of these non-dimensional numbers with Ra_T for different parameter values are depicted in Figs. 3a-c, 4a-c and 5a-c respectively. In Figs. 3a-c, 4a-c and 5a-c it is observed that in each case, nanoparticle concentration Nusselt number *NuF* is always greater than both thermal Nusselt number *NuT* and solute concentration Nusselt number *NuC* Nusselt number and all Nusselt number start with the conduction state value 1 at the point of onset of steady finite amplitude convection. When Ra_T is increased beyond Ra_{Tc}, there is a sharp increase in the values of Nusselt numbers. However further increase in Ra_T will not change *Nu* and *Sh* significantly. It is to be noted that the upper bound of *NuT* is 3 (similar results were obtained by Malashetty et al. [25]). It should also be noted that the upper bound of *NuF* and *NuC* are not 3 (similar results were obtained by Bhadauria et al. [17]). The upper bound of *NuT* remains 3 only for both clear and nanofluid. Whereas, the upper bound for *NuF* and *NuC* for clear fluid is 3 but for nanofluid it is not fixed.

From Figs. 3a and 4a we observe that as the Soret parameter N_{CT} increases, the value of *NuT* and *NuF* decreases, thus showing a decrease in the rate of heat and mass transport, while the Solute concentration Nusselt number *NuC* (Fig. 5a) increases with increase in Soret parameter N_{CT} implying that Soret parameter N_{CT} enhances the Solute concentration Nusselt number. We observe that as the Dufour parameter N_{TC} (Figs. 3b, 4b and 5b) and Solutal Rayleigh number *Rs* (Figs. 3c, 4c and 5c) increases, the value of *NuT*, *NuF* and *NuC* decreases, thus showing a decrease in the rate of heat and mass transport.

The linear solutions exhibit a considerable variety of behavior of the system, and the transition from linear to non-linear convection can be quite complicated, but interesting to deal with. It is needed to study a time dependent results to analyze the same. The transition can be well understood by the analysis of equation (60) whose solution gives a detailed description of the two dimensional problem. The autonomous system of unsteady finite amplitude equations is solved numerically using the Runge-Kutta method. The Nusselt numbers are evaluated as the functions of time *t*, the unsteady transient behavior of *NuT, NuF* and *NuC* is shown graphically in Figs. 6a-g, 7a-g and 8a-g respectively.

These figures indicate that initially when time is small, there occur large scale oscillations in the values of Nusselt numbers indicating an unsteady rate of heat and mass transport in the fluid. As time passes by, these values approach to steady state corresponding to a near convection stage. Figs. (6a, 7a, 8a), (6b, 7b, 8b), (6c, 7c, 8c) and (6d, 7d, 8d) depicts the transient nature of thermal Nusselt number *NuT*, concentration Nusselt number *NuF* number and solute Nusselt number *NuC* on nanoparticle concentration Rayleigh number *Rn*, nanofluid Lewis number *Ln*, modified diffusivity ratio N_A and Solutal Rayleigh number *Rs*. It is observed that as *Rn, Ln*, N_A and *Rs* increases *NuT*, *NuF* and *NuC* increases, thus showing an increase in the heat and mass transport, which are the similar results observed by Agarwal et al. [24]. From Figs. (6d, 7d, 8d) we observe that viscosity ratio ν increases the heat and mass transports and in Figs. (6e, 7e, 8e) we observe that as conductivity ratio η increases the *NuT*, *NuF* and *NuC* decreases indicating that there is retardation on heat and mass transports. Figs. (6g, 7g, 8g) depicts the transient nature of Vadász number *Va*. It is observed that as *Va* increases *NuT*, *NuF* and *NuC* increases, thus showing an increase in the heat and mass transport.

From the figures we can observe that the value of thermal Nusselt number *NuT* starts from 1, the value of nanoparticle concentration Nusselt number *NuF* starts from 6 and the value of solute concentration Nusselt number *NuC* starts form 1.75.

Conclusions

We considered linear stability analysis in a horizontal porous medium saturated by a nanofluid, heated from below and cooled from above, using Darcy model which incorporates the effect of

Brownian motion along with thermophoresis. Further the viscosity and conductivity dependence on nanoparticle fraction was also adopted following Tiwari and Das [20]. Linear analysis has been made using normal mode technique. However for nonlinear analysis truncated Fourier series representation having only two terms is considered. We draw the following conclusions

1. For stationary mode Soret parameter N_{CT} and Dufour parameter N_{TC} have a stabilizing effect while Solutal Rayleigh number *Rs* destabilize the system.

2. For oscillatory mode Soret parameter N_{CT} and Dufour parameter N_{TC} have a stabilizing effect while Solutal Rayleigh number *Rs* destabilize the system.

3. The value of transient Nusselt numbers *NuT* starts from 1, *NuF* starts from 6 and *NuC* starts form 1.75.

4. The effect of time on transient thermal Nusselt number, nanoparticle concentration Nusselt number and solute concentration Nusselt number is found to be oscillatory when *t* is small. However, when *t* becomes very large Nusselt numbers approaches to the steady value.

List of Symbols

C	solute concentration
D_B	Brownian diffusion coefficient (m^2/s)
D_T	thermophoretic diffusion coefficient (m^2/s)
H	dimensional layer depth (m)
k	thermal conductivity of the nanofluid (W/m K)
k_m	overall thermal conductivity of the porous medium saturated by the nanofluid (W/m K)
K	permeability (m^2)
Le	thermo-solutal Lewis number
Ln	Lewis number
N_A	modified diffusivity ratio

N_B	modified particle-density increment
N_{CT}	Soret parameter
N_{TC}	Dufour parameter
p^*	pressure (Pa)
p	dimensionless pressure, $\left(p^*K\right)/\left(\mu\alpha_f\right)$
γ_a	non dimensional acceleration coefficient
Va	Vadász number
Ra_T	thermal Rayleigh- Darcy number
Rm	basic-density Rayleigh number
Rn	concentration Rayleigh number
Rs	solutal Rayleigh number
t^*	time (s)
t	dimensionless time, $\left(t^*\alpha_f\right)/H^2$
T^*	nanofluid temperature (K)
T	dimensionless temperature, $\frac{T^*-T_c^*}{T_h^*-T_c^*}$
T_c^*	temperature at the upper wall (K)
T_h^*	temperature at the lower wall (K)
(u,v,w)	dimensionless Darcy velocity components $\left(u^*,v^*,w^*\right)H/\alpha_m$ (m/s)
v	nanofluid velocity (m/s)
(x,y,z)	dimensionless Cartesian coordinate $\left(x^*,y^*,z^*\right)/H$; z is the vertically upward coordinate
$\left(x^*,y^*,z^*\right)$	cartesian coordinates

Greek symbols

α_f	thermal diffusivity of the fluid, $(\mathrm{m/s^2})$
β_C	solutal volumetric coefficient (K^{-1})
β_T	thermal volumetric coefficient (K^{-1})

ν	viscosity variation parameter
ε	porosity
η	conductivity variation parameter
μ	viscosity of the fluid
ρ	fluid density
ρ_p	nanoparticle mass density
σ	thermal capacity ratio
ϕ^*	nanoparticle volume fraction
ϕ	Relative nanoparticle volume fraction, $\frac{\phi^* - \phi_0^*}{\phi_1^* - \phi_0^*}$

Subscripts/superscripts

b	basic solution
f	fluid
p	particle
*	dimensional variable
'	perturbed variable
St	stationary
Osc	oscillatory

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